LAMINAR HEAT CONVECTION OF A LIQUID IN AN ANNULAR REGION WITH A GIVEN HEAT FLUX

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We consider unsteady flow and heat transfer for a viscous incompressible liquid in a horizontal annular channel with a constant heat flux on its outer surface.

The investigation is based on numerical solution of the system of two-dimensional unsteady equations of motion, continuity, and energy, which has the following form in the polar coordinate system [1]:

$$\frac{\partial f}{\partial \tau} + \frac{1}{r}\left(\frac{\partial F}{\partial \varphi}\frac{\partial f}{\partial r} - \frac{\partial F}{\partial r}\frac{\partial f}{\partial \varphi}\right) = \Pr \nabla^2 f + \mathrm{GrPr}^2\left(\frac{1}{r}\frac{\partial \Theta}{\partial \varphi}\sin\varphi - \frac{\partial \Theta}{\partial r}\cos\varphi\right),$$

$$\frac{\partial \Theta}{\partial \tau} + \frac{1}{r}\left(\frac{\partial F}{\partial \varphi}\frac{\partial \Theta}{\partial r} - \frac{\partial F}{\partial r}\frac{\partial \Theta}{\partial r}\right) = \nabla^2\Theta, \quad f = \nabla^2 F,$$

where F and f are the dimensionless stream function and vorticity, respectively; $r_i = R_i/\delta$ are the dimensionless radii of the inner (i = 1) and outer (i = 2) cylinders; and $\delta = R_2 - R_1$ is the gap between the cylinders.

As a temperature scale for dimensionless temperature Θ we choose the quantity $\langle \Delta T \rangle$, equal to the difference between the average temperatures of the outer and inner surfaces, i.e., $\langle \Delta T \rangle = \langle T_{W_2} \rangle - T_{W_1}$.

We assume that the liquid is motionless at zero time in the annular region and that the temperature distribution corresponds to heat-conduction conditions. A constant temperature $\Theta_{W_1} = T_{W_1}/\langle \Delta T \rangle$ is maintained on the inner cylinder, and a constant heat flux q_{W_2}, is maintained on the outer cylinder, which is equivalent to the condition $(\partial\Theta/\partial r)_{W_2} = 1$ with the assumed scale. The initial distribution $\Theta(r, \varphi)$, in accordance with the heat-conduction equation and the given boundary conditions has the form

$$\Theta = r_2 \ln r/r_1 + \Theta_{w_1}.$$

The following similarity parameters are introduced into the original systems of equations: $\mathrm{Gr} = g\beta \cdot (\langle T_{W_2} \rangle - T_{W_1})/\nu^2$, the Grashof parameter; $\Pr = \nu/a$, the Prandtl number; and $\mathrm{Fo} = \tau = at/\delta^2$, the Fourier number.

The Zeidel method was used for numerical solution of the system of equations of convective heat transfer, following a preliminary integration of the equations in an elementary cell of the mesh [1]; the Poisson equation was solved by the method of variable directions. We used second-order formulas to approximate the derivatives on the boundaries of the region. The computation was carried out in a 17 × 17 mesh for half of the annular region (we assumed symmetry relative to the vertical axis passing through the center of the annular layer). The difference of the main results of the computation from those obtained from a finer 22 × 22 mesh was not more than 3%.

Figures 1 and 2 show the development with time of the circulation motion and the variation of the radial V_r and tangential V_φ velocity components in different sections of the annular region ($\mathrm{Gr} = 10^4$, $\Pr = 0.7$, $r_2/r_1 = 2$ and g is the acceleration due to gravity, m/sec^2). If the liquid is at rest at zero time, then for Fo = 0.02 the velocities reach their maximum values, and then gradually decrease, approaching some constant values (Fo = 1), as can be seen by comparing Figs. 1 and 2. Here the center of vorticity (and also the region of minimum velocity) moves downward as the steady conditions become established.

It is characteristic that with increase of Rayleigh number (Ra = GrPr) there is a gradual decrease in the time at which the maximum convective intensity is reached.

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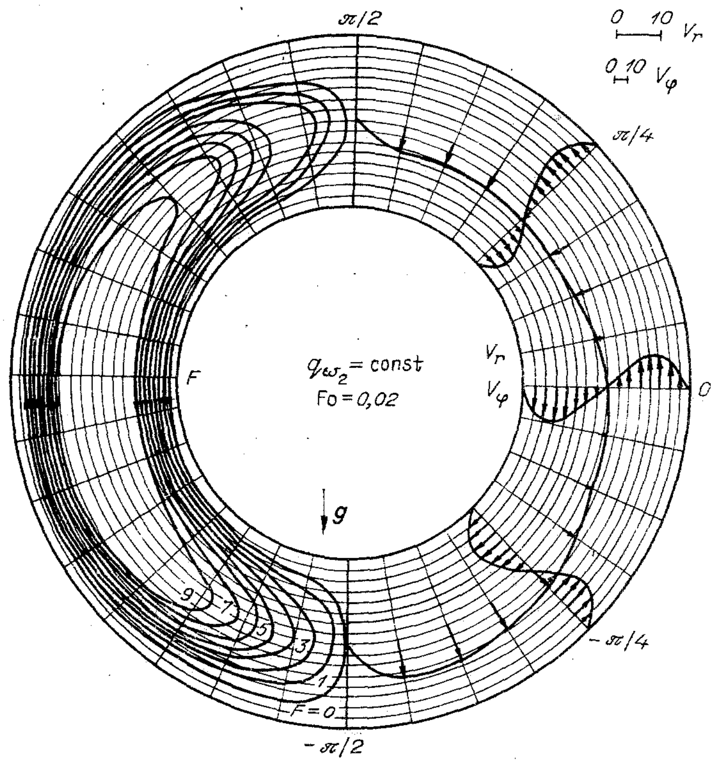

Fig. 1

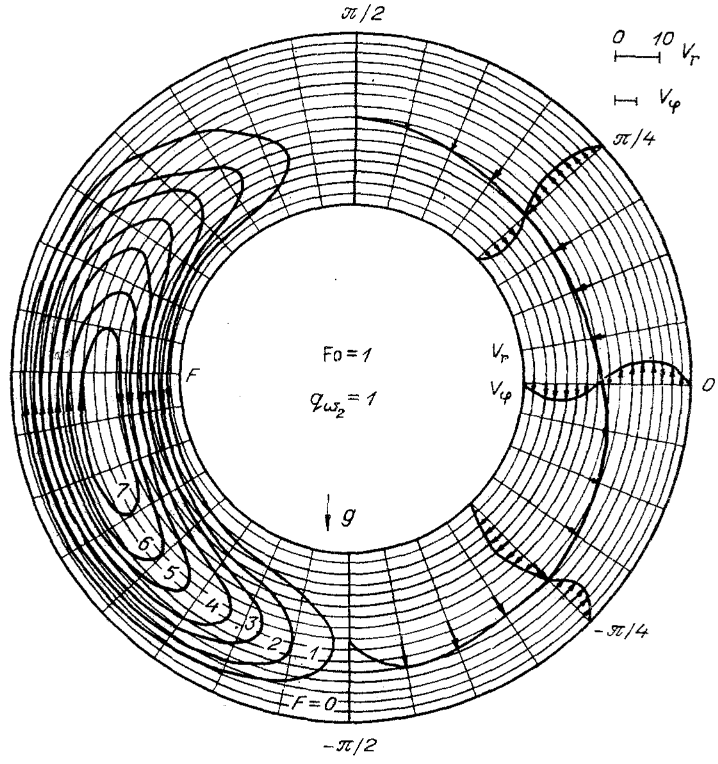

Fig. 2

We can judge the beginning of the influence of convection in the field $\Theta(r, \varphi)$ from the appearance of the vertical temperature differences. Figure 3 shows the variation with time of the temperature distributions for five values of the polar angle φ under conditions given by $Gr = 10^4$, $Pr = 0.7$, $r_2/r_1 = 2$, $q_{w_2} = 1$, where the solid curves correspond to $Fo = 1$ and the dashed curves, to $Fo = 0.02$.

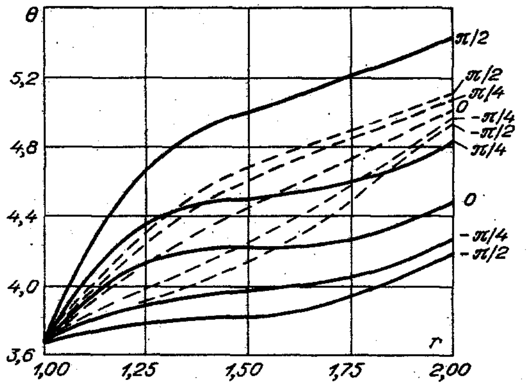

Fig. 3

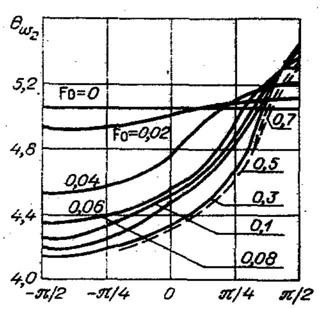

Fig. 4

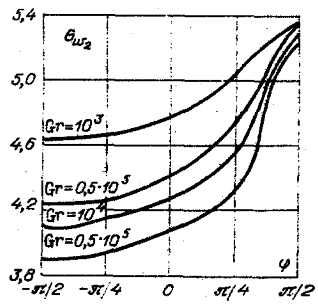

Fig. 5

The one-dimensional temperature field $\Theta = \Theta(r)$ for Fo = 0 begins to experience these convection effects for Fo = 0.02. The temperature profiles begin to diverge and for Fo = 0.1 reach a steady distribution, with temperature layering typical of convection.

A similar relation obtains in the variation of the temperature of the external wall Θ_{w_2}, as a function of Fo (Fig. 4, $Gr = 10^4$, Pr = 0.7, $r_2/r_1 = 2$). The constant temperature Θ_{w_2} for Fo = 0 becomes appreciably nonuniform along φ so Fo increases, and, starting at Fo = 0.3, practically does not vary with time. The maximum scatter in Θ_{w_2} under steady conditions is 25% of the mean value for these conditions ($Gr = 10^4$).

As is shown in Fig. 5 (Pr = 0.7), the nonuniformity of the temperature of the outer cylinder under steady conditions increases with increase in the Grashof number, and this change is basically caused by decrease in the wall temperature in the lower part of the annular layer.

The investigation of the dependence of the temperature field on the Gr and Fo numbers reveals three characteristic regimes: an initial regime, close to the heat conduction region, when $\Theta = \Theta(\text{Fo})$; a transition regime in which convection begins to affect the temperature distribution ($\Theta = \Theta(\text{Gr, Fo})$); and a stationary regime in which there is no dependence of temperature on time ($\Theta = \Theta(\text{Gr})$). This conclusion agrees with the classification of regimes of flow and heat transfer for unsteady convection in a rectangular region, derived in [2].

Knowing the temperature distribution in the flow field, one can calculate the local Nusselt numbers at the boundaries of the region, which are determined in this case in terms of local temperature differences between the outer and inner walls for each value of φ:

$$\text{Nu}_i(\varphi) = \left(\frac{\partial \Theta}{\partial r}\right)_i \frac{1}{\Delta\Theta}, \quad i = 1, 2,$$

while the mean Nusselt number is obtained by averaging the local values with respect to φ in the range $[-\pi/2, \pi/2]$ and is

$$\langle \text{Nu} \rangle_i = \left\langle \frac{\partial \Theta}{\partial r} \right\rangle_i \frac{1}{\langle \Delta\Theta \rangle}.$$

The graphs of variation of $\text{Nu}_i(\varphi)$ on the inner and outer walls for various values of Grashof number are shown in Fig. 6 [1) $Gr = 0.5 \cdot 10^5$; 2) $Gr = 10^4$; 3) $Gr = 0.5 \cdot 10^4$; 4) $Gr = 10^3$; 5) heat-conduction regime with Pr = 0.7, $r_2/r_1 = 2$].

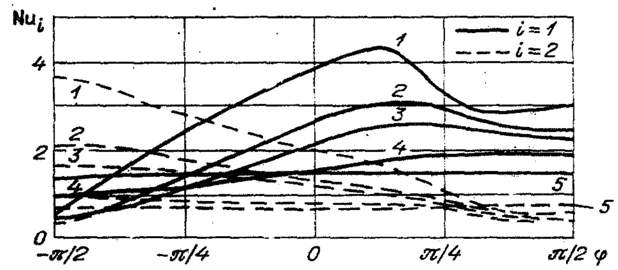

Fig. 6

Following determination of the Nusselt number, the function $Nu_2(\varphi)$ in essence reproduces a curve, the inverse of $\Theta_{w_2}(\varphi)$ (see Fig. 5) at the appropriate Gr number, since $\Theta_{w_2}(\varphi)$ determines the local difference Θ_{w_1}, to an accuracy within a constant $\Delta\Theta(\varphi)$.

With increase in the Gr number from 10^3 to $0.5 \cdot 10^5$, there is a decrease in the local numbers Nu_2 along the flow of hot liquid from $\varphi = -\pi/2$ to $\varphi = \pi/2$, around the outer cylinder wall. At some value of φ in the range $[\pi/4, \pi/2]$ the local values of Nu become less than in the heat-conduction regime, since the temperature difference $(\Theta_{w_2} - \Theta_{w_1})$ in this region under steady conditions becomes less than $\Delta\Theta$ in the heat-conduction regime.

Thus, when there is a constant heat flux along the outer cylinder surface, the distribution of local Nu numbers with respect to φ differs more from the corresponding relation in the purely heat-conduction regime than the variation of Θ_{w_2} along φ (see Fig. 5).

In regard to the family of $Nu_1(\varphi)$ curves, constructed for various Gr values (see Fig. 6), these must be analyzed with allowance for variation of local heat flux $(\partial\Theta/\partial r)_1$ along the inner cylinder surface. With liquid motion downward around the cold cylinder wall the local temperature gradients decrease along φ, because of increase of the boundary-layer thickness. Because these are normalized to different $\Delta\Theta(\varphi)$, the form of the $Nu_1(\varphi)$ curves differs from the analogous relations in the case of isothermal boundaries [1].

This difference begins to show up at $Gr > 10^3$, when the nonuniformity of Θ_{w_2} as a function of φ becomes appreciable. We see a rise in the curves giving Nu_1, as a function of φ for $\pi/2 > \varphi > \pi/8$, in spite of a decrease in local heat flux, which is due to the sharp decrease in the $\Delta\Theta(\varphi)$ drop in this range of φ (see Fig. 5). With further variation of φ from $\pi/8$ to $-\pi/2$, the sharp decrease in $\Delta\Theta(\varphi)$ stops, which leads to a decrease in the Nu number in this range of φ. At a certain value of φ the local Nu numbers become less than in the liquid at rest.

The maxima of the $Nu_1(\varphi)$ curves (see Fig. 6) are displaced with increase of Gr number. This corresponds to a displacement to the right of the point at which the sharp increase in $\partial\Theta/\partial\varphi$ occurs (see Fig. 5).

The sharp variation in the nature of the $\Theta_{w_2}(\varphi)$ curves for $\varphi \approx \pi/4$ is due to the considerable drop in the velocities near this point.

For the characteristics of heat-transfer intensity under various flow regimes, we obtained a convective coefficient dependence given by the formula

$$\varepsilon_{k_i} = \langle \mathrm{Nu} \rangle_i r_i \ln \frac{r_2}{r_1}, \quad i = 1, 2,$$

as a function of Rayleigh number.

During the establishment of steady conditions the dependence $\varepsilon_k = \varepsilon_k(\mathrm{Ra}, \mathrm{Fo})$ converts to $\varepsilon_k = \varepsilon_k(\mathrm{Ra})$ with increase in Fo. The boundary for establishment of steady heat-transfer conditions during convective motion is approximated by the formula $\mathrm{Fo} = 1.52/\mathrm{Ra}^{0.12}$.

Under steady conditions the graphs of ε_k as a function of Rayleigh number is approximated by the parametric relation $\varepsilon_k = 0.257\mathrm{Ra}^{0.21}$.

In comparing with an analogous relation in the case of isothermal walls it is clear that for small Ra ($\mathrm{Ra} < 2300$) the heat transfer is more intense under the condition $q_{w_2} = \mathrm{const}$, while for large Ra it is more intense in the case $\Theta_{w_2} = \mathrm{const}$. This is due to a redistribution of outer wall temperature under conditions of constant heat flux to it and to the formation of a stagnant heated zone in the upper part of the region, preventing heat loss from the hot wall.

LITERATURE CITED

1. G. B. Petrazhitskii, E. V. Bekneva, V. A. Brailovskaya, and N. M. Stankevich, "Calculation of the flow and heat transfer in free motion of a liquid in a horizontal annular channel," Vestn. L'vov. Politekh. Inst., Vopr. Élektro–Teploénerg., No. 46 (1970).
2. V. I. Polezhaev, "Unsteady laminar heat convection in a closed cavity with prescribed heat flux," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1974).

ANALYTICAL INVESTIGATION OF THE SOLIDIFICATION PROCESS OF A LIQUID METAL IN CONTINUOUS CASTING UNITS

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We consider the fully established process of the solidification of a flat continuous ingot in a cooling system with a liquid-metal heat-transfer medium, filling the gap between the surface of the ingot and the water-cooled wall of the crystallizer (Fig. 1, where 1 is the ingot; 2 is the liquid-metal heat-transfer medium; 3 is the wall; 4 is the cooling medium; and 5 is a capillary packing). Here we shall assume that the external water cooling can be regulated along the ingot, for example, by sectional heat removal. The presence of a liquid-metal heat-transfer medium between the surface of the ingot and the water-cooled wall excludes the formation of a gas gap, which makes it possible to increase the rate of the cooling process, making it uniform around the perimeter of the ingot.

We assume that the transfer of heat along the Z axis due to thermal conductivity can be neglected in comparison with convective heat transfer [1] and that the temperature of the metal in the liquid phase is equal to the crystallization temperature. Under these conditions, we shall take account of the effect of heating of the melt by a corresponding increase in the latent heat of fusion in the approximation of the Stefan condition.

§1. If the width of the ingot is much greater than its thickness, then the solution of the problem posed will depend only on the two variables x and z. We select a Cartesian system of coordinates with the Z axis lying in the plane of symmetry of the ingot and as the origin of coordinates we take the point of intersection of the Z axis with a plane passing through the point of the start of crystallization. Taking account of the assumptions made above, the equation determining the temperature distribution in the solid phase has the form

$$\rho v C \frac{\partial T}{\partial z} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right), \tag{1.1}$$

where v is the velocity; ρ is the density; C is the heat capacity; and λ is the thermal conductivity of the ingot.

We write the boundary condition at the surface of the ingot in the form of the Newton–Richman law

$$\lambda \frac{\partial T}{\partial x}\Big|_{x=x_0} = -k\left(T|_{x=x_0} - T_M(z)\right), \tag{1.2}$$

where $2x_0$ is the thickness of the ingot; T_M is the temperature of the cooling medium (water), which is assumed to be a given function of the coordinate z; $k = (R_H + R_W + R_r)$ is the heat-transfer coefficient; R_H and R_W are the thermal resistances of the liquid-metal heat-transfer medium and the wall; and R_r is the external heat resistance.

At the crystallization surface, the following conditions must be observed:

$$\lambda \frac{\partial T}{\partial x}\Big|_{x=\xi(z)} = \varkappa^* \rho_c v \xi'(z); \tag{1.3}$$

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